Towards Iterative Combinatorial Exchanges

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Motivation

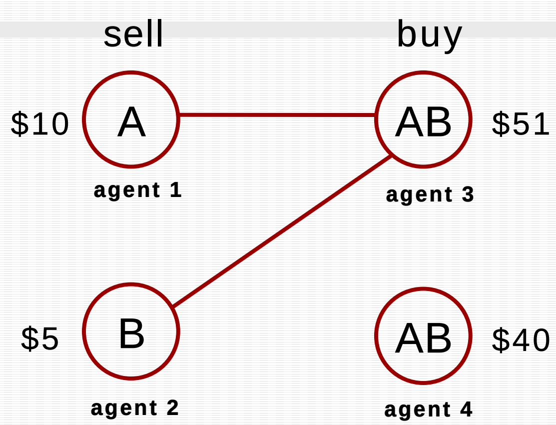
- Highly fragmented spectrum (frequency, control, and geography)
  - result of administrative allocation
- 2.5-2.7 GHz Spectrum
  - more total spectrum than cellular and PCS
  - 19@ 6MHz Instructional TV
  - 12 @ 6MHz MDS (wireless cable)
  - 493 Basic Trading Areas
- A "big bang" exchange:
  - make large amounts of spectrum (assigned & unassigned) available
  - improve allocative efficiency, take advantage of new technologies

Combinatorial Exchanges

- Multiple buyers and sellers, w/ expressive bids
  - e.g. "Buy {NYC, PHL, BOS} and sell {DC} for $1million"
- FCC can also participate, actively:
  - e.g. "Convert ITFS licenses into wireless phone licenses"

  and passively (define aggregations):
  - e.g. "all contiguous 6 MHz blocks of spectrum in a BTA are equivalent"
  - may help computationally, and mitigate hold-out problem

Stylized Example.



Efficient trade: 1 and 2 sell, 3 buys. Surplus $51.

Main Challenges

- Winner-determination
  - likely to be harder than one-sided auctions (Sandholm's talk)
- Economic
  - mitigating the bargaining or "hold-out" problem
- Preference elicitation
  - hard valuation problems
  - iterative designs likely important to guide elicitation

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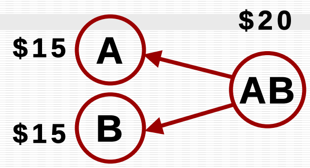
Bargaining Problem.

- Example:



- Many *ex post* Nash equilibrium:
  - ($5,$15,$20); ($10,$10,$20); ($15,$15,$30)...
  - presents an efficiency problem, because agents need to select an equilibrium.
- Construct *ex post* Nash:
  - allocate π^t to agent i with $V^t(N)-V^t(N\backslash i)>0$
  - adjust $V^{t+1}(N)$, $V^{t+1}(N\backslash i)$ to $v^{t+1}=\max(0,v^t-\pi^t)$
  - repeat.

A One-Shot Design

(Parkes, Kalagnanam and Eso, 2001)

- Collect bids
- Compute $V(N)$, value of surplus-maximizing trade given all bids
  - implement this outcome
- Compute $V(N\backslash i)$, value of surplus-maximizing trade without bids from i
- Divide surplus $\sum_i \pi_i = V(N)$ across participants; try to mitigate bargaining problem

Surplus Division

Payoffs $\pi_i \geq 0$ to solve:

$$\min_\pi f(\pi)$$

s.t. $\sum_i \pi_i \leq V(N)$ (BB)

$\pi_i \leq V(N)-V(N\backslash i), \forall i$ (*)

$\pi_i \geq 0, \forall i$ (P)

Note1: $\pi_{VCG,i}=V(N)-V(N\backslash i)$

Note 2: (BB) and $\sum_{j\neq i}\pi_j \geq V(N\backslash i)$ (1-core) is equivalent to (*)

Lemma. Any mechanism satisfying (BB), (*), and (P) has *ex post regret* $\pi_{VCG,i}$ for agent i given bids of other agents

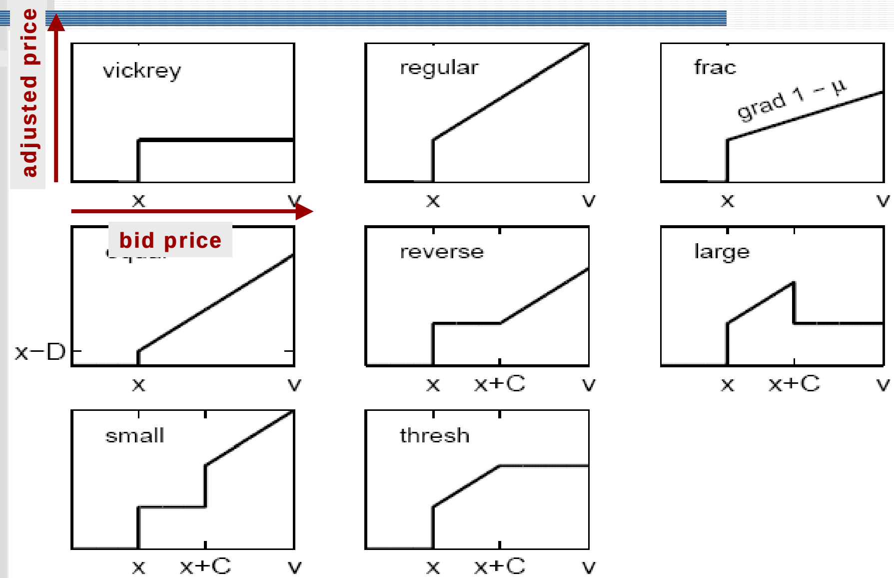
VCG-Based Schemes

Somewhat natural to consider VCG-based schemes, that divide surplus according to *ex post* π_{VCG} ("payoff left on table")

- Threshold. Minimize worst-case $(\pi_{VCG,i}-\pi_i)$
- Fractional. Each agent gets $\pi_i=\mu\ \pi_{VCG,i}$
- Large. Allocate payoff in order $\pi_1, \pi_2, \pi_3\ldots$
- Reverse....

Threshold scheme minimizes the maximal ex post regret across all agents.

Stylized Representations



Threshold Rule

- Implements a slight variation of the k-DA uniform price, double auction with $k=0.5$ (Wilson'85)
  - Threshold payoff division implemented with price $p^*=0.5(\min(a_{k+1},b_k)+\max(b_{k+1},a_k))$, asks $a_1<a_2<\ldots<a_m$, bids $b_1>b_2>\ldots b_m$, k items trade
- Second-best (for efficiency) for the standard single item bargaining problem, for i.i.d. Uniform [0,1] values and costs
  - linear-strategy equilibrium; with $\hat{v}=(2/3)v+1/12$ and $\hat{c}=(2/3)c+1/4$ (Myerson & Satterthwaite, 83)

Experiments: *ex ante* BNE

- Consider a very limited strategy space:
  - $b_i(S)=(1-\alpha)\ v_i(S),\ \forall\ S$, if buyer
  - $b_i(S)=(1+\alpha)v_i(S),\ \forall\ S$, if seller

- Compute a symmetric *ex ante* BNE:

$$\alpha^* = \arg\max_\alpha E_i\ E_{-i}\left[v_i(x^*(b))-\ p_i(b)\right]$$

where $x^*(b)$ is allocation given bids b, price $p_i(b)$ is payment by agent i, and the expectation is taken w.r.t. distribution over types of agents.

Naive Approach

- Enumerate a payoff matrix, compute *ex ante* BNE

α_{-i}

		-0.5	-0.48	…	0	…	0.12	0.14	…	0.98	1.0
	-0.5	1.5	1.4								
	-0.48	1.3									
	…										
	0										
	…										
α_i	0.12										
	0.14										
	…										
	0.98										
	1.0										

Took 2.5 days, for a grid size of 0.01, 500 instances, 5 buyers, 5 sellers, 20 goods, 10 bids/asks per agent.

Algorithm

(w/ David Kyrch)

- Choose a small set of strategies $A^0=(\alpha_1,\ldots,\alpha_M)\in [a,b]^0$, for some initial search space.
- Assume all agents except agent 1 play α^t; initialize to α^0.
- Compute the BR, $\alpha^*\in A^t$, given average payoff to agent 1 and α^t
- Move α^{t+1} towards α^*; refine $[a,b]^{t+1}$ to focus search, select new A^{t+1}.

Details.

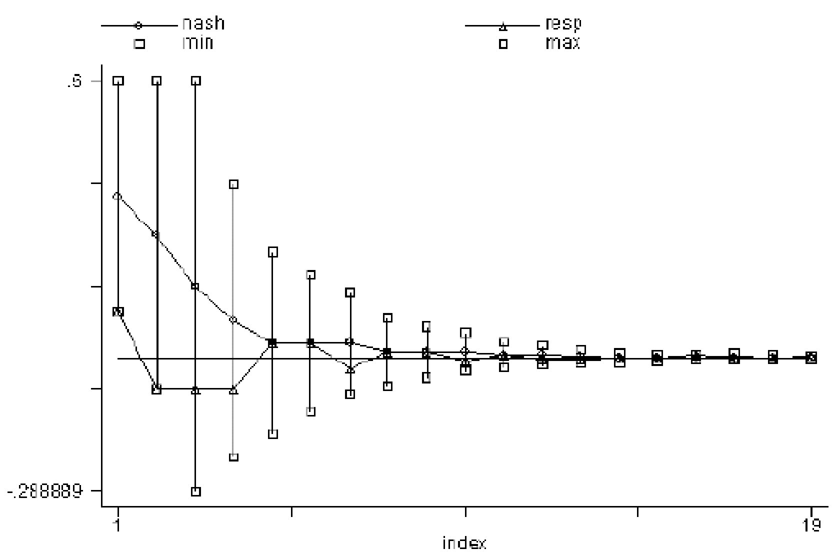
- new center: $\alpha^{t+1}=1/3\ \alpha^* + 2/3\ \alpha^t$
- $|A|=7$
- terminate when α^{t+1} is within 0.01 of α^t
- new range: $[a,b]^{t+1}$ centered on α^{t+1}
  - $(b\text{-}a)^{t+1}=3/4\ (b\text{-}a)^t$, if $\alpha^{t+1}\in [a,b]^t$
  - $(b\text{-}a)^{t+1}=4(\alpha^{t+1}-\alpha^*)$, otherwse.

- Final validation step:
  - discretize $[a,b]^0$ to grid level 0.01, and check α^* is a BR to α^*_{-i}.

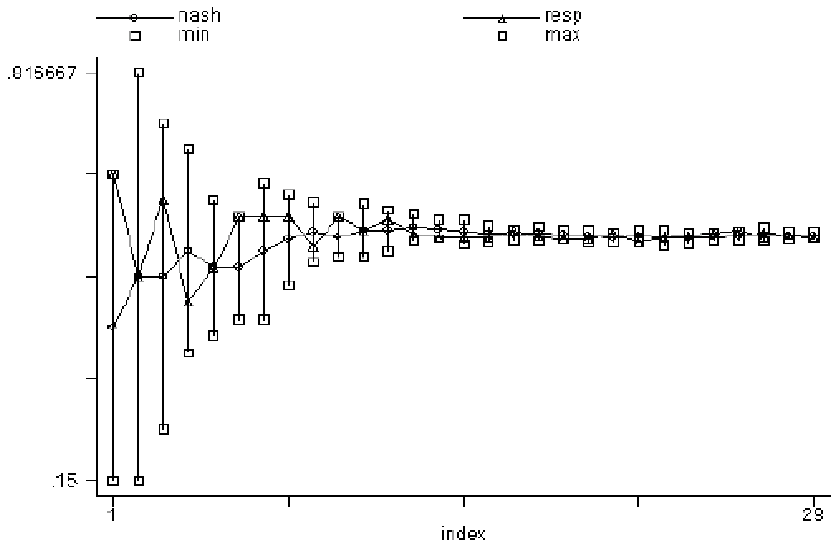
Experimental Results

- 5 buyers, 5 sellers, 20 goods
- 10 bundles/ agent.
  - Generate using Uniform (Sandholm'99); with value U[0,1], same # items (drawn at random) in each bundle.
  - assume XOR logic between bundles
- Winner determination performed by commercial MIP software.
- Evaluate over 500 instances.
- Speed-up: 1% accuracy in 2.5 CPU hours (c/f 2.5 days for enumeration)

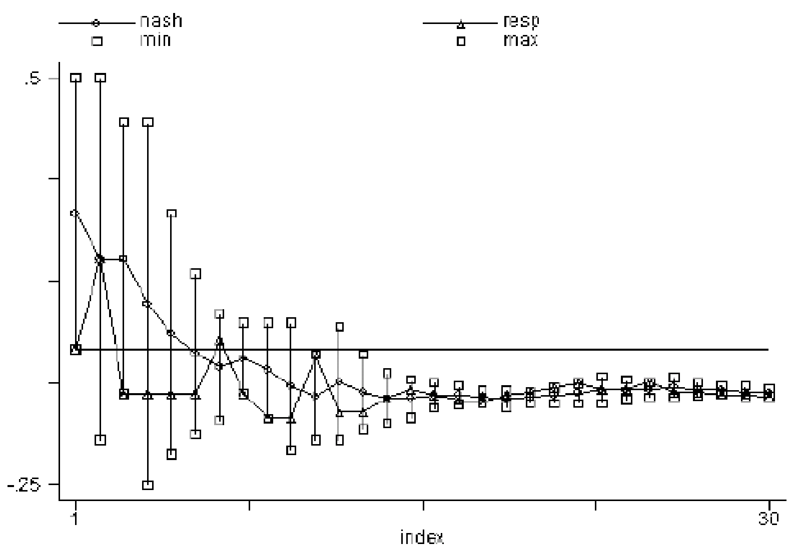
Example 1- VCG payments



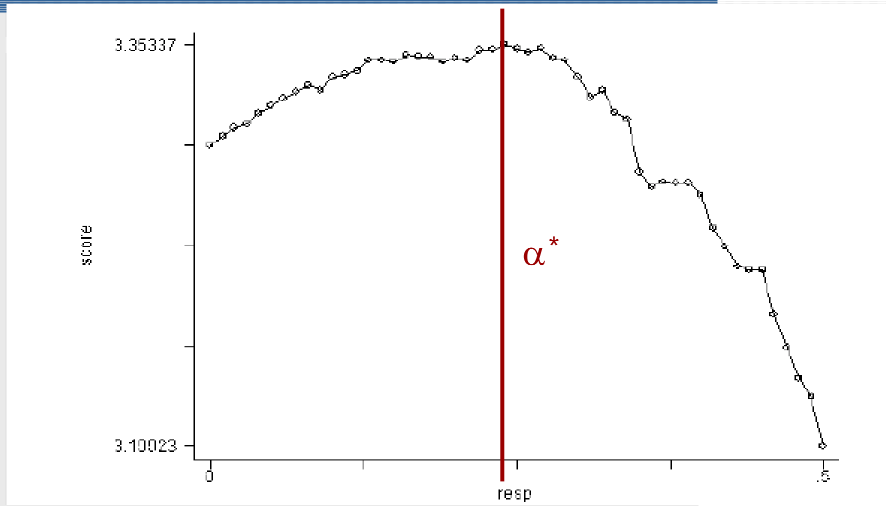
Example 2- No Discount



Example 3- Large

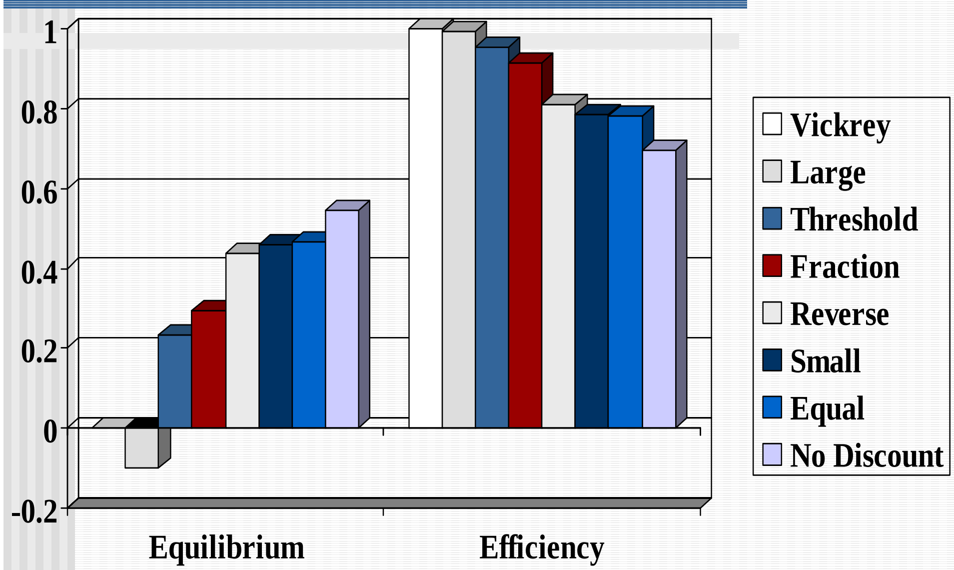


Example: Validity



Validating *ex post* Nash of Threshold rule

Main Results



Large rule

- $\alpha^* \approx$-0.08
- Optimal strategy is to overbid, and win $\pi_i = \pi_{VCG,i}$
  - Implies that at least one participant has negative ex post payoff in BNE
- Unstable to high bids:
  - a buyer can always benefit from overstating her vaue if she knows she will win

NB. d'AGVA "expected Groves" mechanism is BB and *ex post* EFF, but only *ex ante* IR. (also needs an informed designer) (Arrow'79,d'Aspremont & Gerard-Varet'79)

Threshold Scheme

- Threshold is most efficient after Large (95% vs. 99%)
- Threshold rule is more stable:
  - buyers cannot benefit from overstating values (quite general assumptions)
  - a buyer, i, receiving payoff division π_i, can only benefit from decreasing its bid (by less than π_i) when there is some $V(N\backslash j)$, $j \neq i$, without i.
- Also, reduces nicely to existing double auctions in special cases.

FCC: A Special Player (Milgrom)

- Can also apply core constraints for the FCC
  - $\pi_{FCC} + \sum_{i\in L}\pi_i \geq V(FCC \cup L),\ \forall\, L \subseteq (N \setminus FCC)$
- Nice alternative to involving the FCC as an active bidder.

$\Rightarrow$ the FCC cannot make more revenue simply by looking at the outcome

- cannot propose an alternative with more revenue that a subset of participants will all prefer (based on their reports).

Main Challenges

- Winner-determination
  - likely to be harder than one-sided auctions
- Economic
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- Preference elicitation
  - hard valuation problems
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Preference Elicitation

- One-sided auctions
  - ascending-price auctions (linear prices, non-linear prices, non-linear & anonymous prices)
  - proxy agents (decouple best-response queries from user queries)
  - direct elicitation approaches (query-based, e.g. value queries, ordinal queries, etc.)

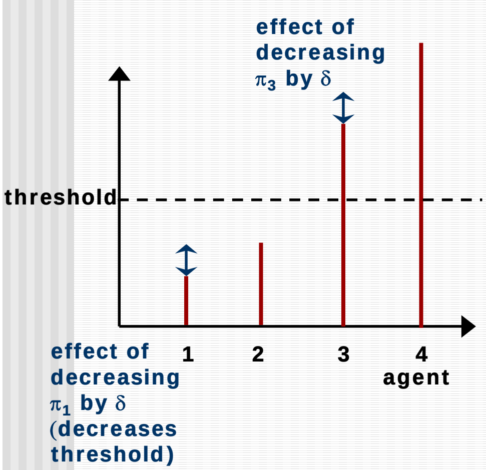
Elicitation for Exchanges: Key Problems.

- Item discovery
  - scope of exchange may not be initially known
- Price discovery
  - may be *no trade* in initial stages
- Bargaining
  - the bargaining problem is omnipresent
  - not present in one-sided auctions when VCG outcome in core.

High-Level Approach.

- Proxied
  - users make *direct* but *incremental* statements about valuations for different bundles.
- Threshold-based.
  - solve WD to maximize reported surplus, and implement the Threshold payoff-division rule
- Activity Rules.
  - consistency: incremental value information must not contradict with earlier information.
  - require “progress” across stages.
- Staged w/ Final Round.
  - price-based feedback

Information Required.



Consider information:

$v'_i(S)=\max(0,v_i(S)-\pi_i)$

Can compute VCG, if:

1. Complete info from all losers
2. high enough bids from winner i to place them in $V'(N\backslash j)$, for all $j\neq i$, or complete info if not possible for any $\pi_i>0$.

For Threshold, also need:

3. high enough bids from winner i to gain some payoff division, or complete info if not possible for any $\pi_i>0$

Proxy Information: Upper & Lower Bounds

- Agents provide *bounds* on values of bundles
  - $v_i(S) \in [\underline{b}_i(S), \overline{b}_i(S)]$
  - Can be via a compact bidding language ("upper-bid" and "lower-bid")
  - Maintain consistency, w/ $\underline{b}(S') \geq \underline{b}(S), \forall S' \supseteq S$; $\overline{b}(S') \leq \overline{b}(S), \forall S' \subseteq S$
- Refine bounds between stages, and introduce new bundles
- "Relaxed→Tight" information allows early price discovery

Activity Rule (Flavors).

- Consistency:
  - can refine bounds on existing bundles
  - new bounds within existing bounds
- Progress:
  - tighten limits on allowed slack between bounds in later stages
  - limit # of additional bundles that can introduced in later stages
- At some point, move to a final stage.

In Each Stage…

- Compute Threshold outcome w/ high bids and low asks
  - "high" outcome
  - provides feedback in early stages
- Compute Threshold outcome w/ low bids and high asks
  - "low" outcome
  - provides feedback in later stages
  - **finally implement this outcome**

Price-Feedback/ Buy-side

- Compute high bid prices $\bar{p}_{bid,j}$ for items j based on high bids $\bar{b}_i(S)$

$$\min_{p,\delta} \delta$$

$$\text{s.t. } \bar{b}_i(S') \geq \sum_{j \in S'} \bar{p}_{bid,j} \quad \forall \text{ winner i, winner S'}$$

$$\bar{b}_i(S) \leq [\bar{b}_i(S') - \sum_{j \in S'} \bar{p}_{bid,j}] + \delta + \sum_{j \in S} \bar{p}_{bid,j} \quad \forall \text{ winner i, loser S}$$

$$\bar{b}_i(S) \leq \delta + \sum_{j \in S} \bar{p}_{bid,j}, \quad \forall \text{ loser i}$$

(assumes an XOR bidding language, use Kohlberg (72) iterative scheme to define unique solution, might also want to do *smoothing* across stages.)

- Provide accurate winner feedback, suggest how far can drop price and still win.

Price-Feedback/ Buy-side

- Compute low bid prices $\underline{p}_{bid,j}$ for items j based on low bids $\underline{b}_i(S)$

$\min_{p,\delta} \delta$

s.t. $\underline{b}_i(S) \leq \sum_{j \in S} \underline{p}_{bid,j} \quad \forall$ loser i

$\underline{b}_i(S') \geq \sum_{j \in S'} \underline{p}_{bid,j} - \delta \quad \forall$ winner i, winner S'

$\underline{b}_i(S) \leq [\underline{b}_i(S') - \sum_{j \in S'} \underline{p}_{bid,j}] + \delta + \sum_{j \in S} \underline{p}_{bid,j} \quad \forall$ winner i, loser S

- Provide accurate loser feedback, suggest how far must increase price to win.

Price Feedback/ Sell-side

- Compute low ask prices $\underline{p}_{ask,j}$ to give winner feedback, suggest how far can increase price and still win
  - make these prices accurate for winners, with $\underline{b}_i(S') \leq \sum_{j \in S'} \underline{p}_{ask,j}$, $\forall$ winners (i, S')

- Compute high ask prices $\overline{p}_{ask,j}$ to give loser feedback, suggest how must drop price to win
  - make these prices accurate for losers, with $\overline{b}_i(S) \geq \sum_{j \in S} p_{bid,j}$, $\forall$ losers i

Item Discovery.

- Also need buy-side prices for items offered on sell-side
  - perhaps $0.5(\underline{p}_{ask,j} + \bar{p}_{ask,j})$ is a good signal?
- Also need sell-side prices for items requested on buy-side
  - perhaps $0.5(\underline{p}_{buy,j} + \bar{p}_{buy,j})$ is a good signal?

Next steps.

1
- Put together a computer-based simulation of this system.
- Implement simple bidding agents, check for bad behaviors, refine.

2
- Implement more sophisticated bidding agents, check for bad behaviors, refine.
- Work on computational properties, provide scalability.

3
- Run in an Experimental Economics Lab?

Conclusions.

- A combinatorial exchange can facilitate a "big bang" spectrum auction; allow incumbnents and new entrants to trade
- Key issues are:
  - computational
  - economic (bargaining problem)
  - preference elicitation
- Proposed a straw-model design, lots of interesting questions going forward!