CS286r Iterative Combinatorial Exchanges
Homework 1: Game Theory

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Prof. David Parkes
Division of Engineering and Applied Sciences
Harvard University

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Due: Tuesday 2/17/2004, in the beginning of class. You may use any sources that you want, but you must cite the sources that you use. You can also work in a group, just list off the people you're working with. **If you took the class last year please turn in a brief paper review on one of the papers that will be distributed later this week instead.** Please work hard on making the proofs clear, concise, and easy to read.

1. (10 pts) In the following strategic-form game, what strategies survive iterated elimination of strictly-dominated strategies? What are the pure-strategy Nash eq.?

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

2. (10 pts) Agents 1 and 2 are bargaining over how to split a dollar. Each agent simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the agents receive the shares they named; if $s_1 + s_2 > 1$, then both agents receive zero. What are the pure strategy equilibria of this game?

3. (5 pts) Show that there are no (non-trivial) mixed-strategy Nash eq. (i.e. with support greater than one) in the Prisoners' Dilemma game.

Prisoners' Dilemma

	C	D
C	1,1	-1,2
D	2,-1	0,0

4. (5 pts) Solve for the mixed-strategy Nash eq. in the game in Problem 1.

5. (15 pts) *Battle of the Sexes.* Pat and Chris must choose to go for dinner or go to the movies. Both players would rather spend the evening together than apart, but Pat would rather the go for dinner, and Chris would rather they go to the movies.

		Chris	
		Dinner	Movie
Pat	Dinner	2,1	0,0
	Movie	0,0	1,2

(a) What are the two pure-strategy Nash equilibria?
(b) Let $(q, 1-q)$ be the mixed strategy in which Pat plays Dinner with prob. q, and let $(r, 1-r)$ be the mixed strategy in which Chris plays Dinner with prob. r. Determine the best-response correspondences $q^*(r)$ and $r^*(q)$, and use a similar graphical method to that in class for the "matching pennies" game to determine the mixed-strategy Nash equilibrium.

6. (10 pts) Prove that if strategies, $s^* = (s_1^*, \ldots, s_I^*)$, are a Nash eq. in a strategic-form game $G = \{S_1, \ldots, S_I; u_1, \ldots, u_I\}$, then they survive iterated elimination of strictly dominated strategies. **(hint)** By contradiction, assume that one of the strategies in the Nash eq. is eliminated by iterated elimination of strictly dominated strategies.

7. (15 pts) Prove that if the process of iterated elimination of strictly dominated strategies in game $G = \{S_1, \ldots, S_I; u_1, \ldots, u_I\}$ results in a *unique* strategy profile, $s^* = (s_1^*, \ldots, s_I^*)$, that this is a Nash eq. of the game. **(hint)** By contradiction, suppose there exists some agent i for which $s_i \neq s_i^*$ is preferred over s_i^*, and show a contradiction with the fact that s_i was eliminated.

8. (15 pts) Each of two players receives a ticket on which there is a number in some finite subset S of the interval [0,1]. The number indicates the size of a prize that he might receive. The prizes are identically and independently distributed, with distribution function F. Each player is asked independently and simultaneously whether he wants to exchange his prize for the other player's prize. If both players agree then the prizes are exchanged; otherwise each player receives his own prize. Each player's objective is to maximize his expected payoff. Model this situation as a Bayesian game and show that in any Bayesian-Nash equilibrium the highest prize that either player is willing to exchange is the smallest possible prize.